

# Coupling of whispering-gallery modes in size-mismatched microdisk photonic molecules

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Mechanisms of whispering-gallery (WG)-mode coupling in microdisk photonic molecules (PMs) with slight and significant size mismatches are numerically investigated. The results reveal two different scenarios of modes interaction depending on the degree of this mismatch and offer new insight into how PM parameters can be tuned to control and modify WG-mode wavelengths and  $Q$  factors. From a practical point of view, these findings offer a way to fabricate PM microlaser structures that exhibit low thresholds and directional emission and at the same time are more tolerant to fabrication errors than previously explored coupled-cavity structures composed of identical microresonators. © 2007 Optical Society of America

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During the past decade, photonic molecules (PMs) [1] (clusters of electromagnetically coupled optical microcavities) have come a long way from being a useful illustration of parallels between behaviors of photons and electrons and now hold the promise of new insights into the physics of light-matter interactions and also of a variety of practical applications, including microlasers, tunable filters and switches, coupled-cavity waveguides, and sensors [2–10]. The simplest PM, composed of two identical optical microcavities, has been widely used as a test bed to demonstrate shift and splitting of cavity modes and formation of a spectrum of bonding and antibonding PM supermodes [1–4]. I have recently shown how arranging identical whispering-gallery (WG)-mode microdisks into predesigned high-symmetry configurations yields quasi-single-mode PMs with dramatically increased  $Q$  factors [6], enhanced sensitivity to environmental changes [7], and/or directional emission patterns [8]. In all of these structures, the size uniformity of microcavities is an important issue in successful realization of PM-based optical components.

The motivation for studying interactions of optical modes in a photonic molecule with size mismatch [9,10] stems from two sources. First, precise and repeatable fabrication of identical microcavities, which in many cases are tiny structures just several micrometers in diameter, is highly challenging. Second, a systematic study of double-cavity PMs with various degrees of cavity size mismatch can reveal new mechanisms of manipulating their optical properties, thus paving the way to improving or adding new functionalities to PM-based photonic devices. Such a study has never been performed before to my knowledge and is the focus of this Letter. Despite its simplicity, the double-cavity structure provides useful insight into the general mechanisms of WG-mode coupling and offers new design ideas for more complex structures. The framework of the Muller boundary integral equations previously developed by the author to model PMs composed of identical cavities [7] has been modified to study size-mismatched PMs.

In the following, the term “microcavity mode” encompasses a complex value of the mode eigenfrequency and the corresponding eigenvector (i.e., modal spatial field distribution).

The PM under study is composed of a pair of side-coupled microdisks of radii  $R_A$ ,  $R_B$  and refractive indices  $n_A$ ,  $n_B$  separated by an air gap of width  $w$  [Fig. 1(a)]. Microdisks with thicknesses much smaller than their diameters are considered. Thus, instead of the 3D boundary value problem for the Maxwell equations, I solve its 2D equivalent. In the following analysis, I search for the TE (transverse electric) WG modes, which are dominant in thin disks. At wavelength  $\lambda = 1.521 \mu\text{m}$ , a 2D cavity with radius  $1.1 \mu\text{m}$  and effective refractive index  $n_{\text{eff}}^{\text{TE}} = 2.63$  (2D equivalent of a 200 nm thick GaInAsP disk [2]) supports the  $\text{WG}_{8,1}$  mode with one radial field variation and eight azimuthal field variations ( $Q = 5243$ ). This mode (like all other WG modes in circular cavities) is double-degenerate owing to the symmetry of the structure.

WG-mode degeneracy is removed if two (or more) cavities are brought close together [1–9], and four nondegenerate WG supermodes of different symmetry appear in the double-disk PM spectrum instead of every WG-mode of an isolated cavity. Figures 2(b) and 2(c) show the wavelength migration and  $Q$  factor change of these modes with the change of the radius of one of the cavities. The modes are labeled according to the symmetry of their field patterns along the  $y$  and  $x$  axes, respectively (e.g., the OE supermode has odd symmetry with respect to the  $y$  axis and even symmetry with respect to the  $x$  axis). OE and OO modes are termed antibonding modes, while EO and EE modes are termed bonding ones. Bonding and antibonding supermodes group into nearly degenerate doublets as seen in Fig. 1(b). The values of real parts of eigenfrequencies of two modes in a doublet are so close to each other that they cannot be distinguished [Fig. 1(b)], while their imaginary parts differ, resulting in different  $Q$  factors of these supermodes [Fig. 1(c)]. Thus, in practice only two peaks are observed in

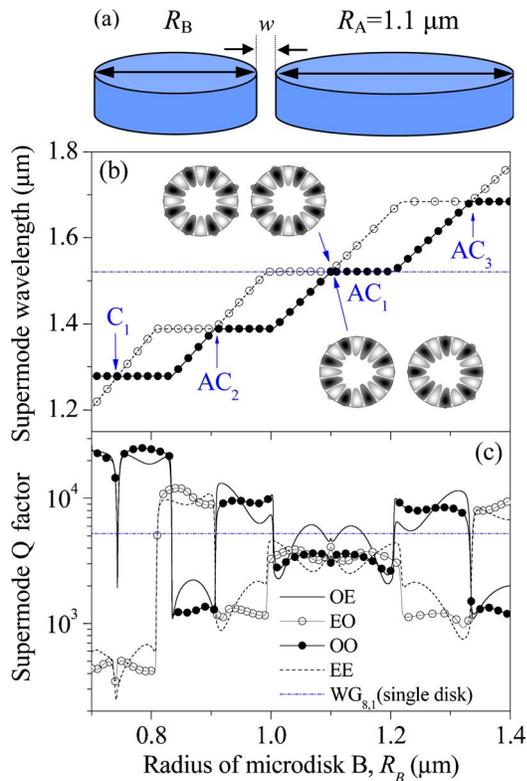


Fig. 1. (Color online) (a) Geometry of a PM composed of two microdisks of different radii; (b) wavelength migration and (c)  $Q$  factor change of PM supermodes as a function of the radius of disk B ( $R_A = 1.1 \mu\text{m}$ ,  $w = 400 \text{ nm}$ ). The insets show the magnetic field distribution of the bonding (EE) and the antibonding (OE)  $\text{WG}_{8,1}$  supermodes in the symmetrical ( $R_A = R_B = 1.1 \mu\text{m}$ ) PM. Here and hereafter, corresponding characteristics of the  $\text{WG}_{8,1}$  mode of an isolated cavity with radius  $1.1 \mu\text{m}$  are plotted for comparison (dashed-dotted line).

a symmetrical double-cavity PM lasing spectrum (see Fig. 2 of Ref. [9]), where the narrow high-intensity peak corresponds to the high- $Q$  antibonding mode doublet and the wider low-intensity peak corresponds to the bonding one.

Careful study of Figs. 1(b) and 1(c) reveals a number of so-called exceptional points (corresponding to certain values of the varied parameter), where PM

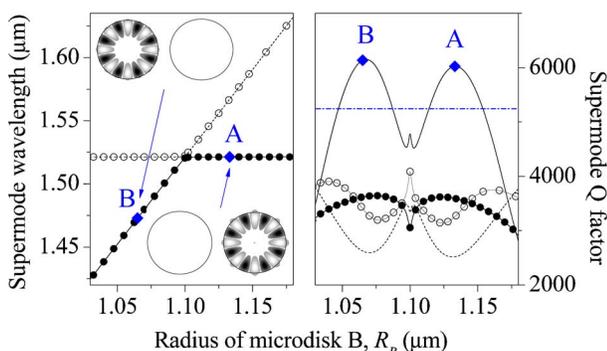


Fig. 2. (Color online) Supermode wavelengths (left) and  $Q$  factors (right) in the vicinity of anticrossing point  $\text{AC}_1$  ( $R_B = R_A$ ). Mode switching (see the modal near-field distributions at points A and B shown in the insets) and  $Q$ -factor enhancement of one of the supermodes can be observed.

supermodes couple following either crossing (C) or avoided crossing (AC) scenarios. The behavior of wavelengths and  $Q$  factors of the four supermodes in the vicinity of these exceptional points is shown in more detail in Figs. 2–4 (for points  $\text{AC}_1$ ,  $\text{AC}_2$ , and  $\text{C}_1$ ). The phenomenon of coupling complex eigenvalues of matrices dependent on parameters when those parameters are changing is of a general nature and is observed in many physical systems [11]. Usually, frequency anticrossing (crossing) is accompanied by crossing (anticrossing) of the corresponding widths of the resonance states. Furthermore, at the points of avoided frequency crossing (points  $\text{AC}_1$ – $\text{AC}_3$  in Fig. 1) eigenmodes interchange their identities, i.e.,  $Q$  factors and field distributions.

In the context of coupling WG-modes in microcavities, this interchange offers exciting new prospects for manipulating the PM optical characteristics, e.g., for realization of optical flip-flops [9]. For example, the coupling of modes with the avoided frequency crossing scenario observed in Figs. 2 and 3 makes switching field intensities between two microdisks possible. To realize such switching in practice, carrier-induced refractive index change of one of the disks induced by nonuniform pumping can be used. This effect was observed experimentally [11] in a PM composed of nearly identical microdisks (similar to the case shown in Fig. 2). If the microcavities are severely size mismatched, their WG modes couple with the frequency crossing scenario. This situation is demonstrated in Fig. 4, and the numerical data indicate that such coupling may significantly spoil the  $Q$  factors of the high- $Q$  modes in the larger microdisk. However, optical coupling between cavities with strongly detuned WG modes makes possible broad spectral transmission effects in coupled-resonator optical waveguides (CROWs) [10], coupled-resonator-induced transparency [12], and significant reducing of CROW bend radiation losses [13].

Finally, enhancement of the  $Q$  factor of a single supermode in a double-disk PM in comparison to its single-cavity value can be observed in Fig. 2 over a relatively wide range of cavity radius detunings:

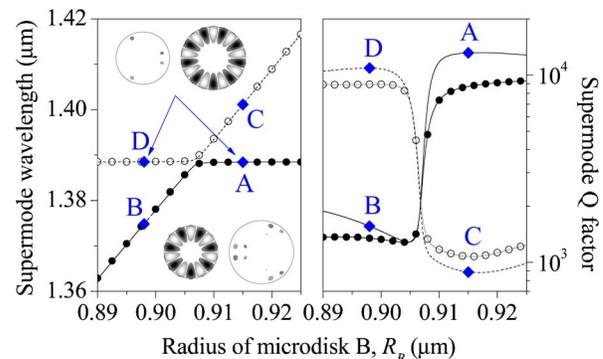


Fig. 3. (Color online) Supermode wavelengths (left) and  $Q$  factors (right) in the vicinity of anti-crossing point  $\text{AC}_2$ . Wavelength repulsion accompanied by the linewidth crossing is observed. The insets demonstrate mode switching. The modal near-field distributions shown in the upper (lower) insets correspond to the eigenfrequency values at the points labeled A and D (B and C), respectively.

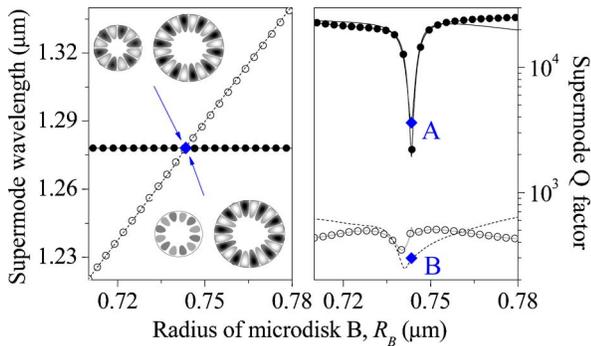


Fig. 4. (Color online) Supermode wavelengths (left) and  $Q$  factors (right) in the vicinity of crossing point  $C_1$ . Wavelength crossing accompanied by damping of the high- $Q$  supermodes is observed. The insets show supermode near-field portraits at the crossing point.

$14 \text{ nm} < \Delta R < 53 \text{ nm}$  ( $\Delta R = |R_A - R_B|$ ). Note that all the other PM supermodes have significantly lower  $Q$  factors in this range of parameter change. This effect offers a way for selective enhancement of the  $Q$  factor of a single supermode that (unlike the symmetry-enhanced  $Q$  factor boost in polygonal PMs) [6,7] does not rely on exact cavity size matching. A possible realization of a PM-based structure designed by using this mechanism of selective mode enhancement is presented in Fig. 5. It consists of three coupled microcavities, with the central cavity radius detuned by  $\Delta R = 35 \text{ nm}$  from the side cavity radii. By adjusting the width of the air gaps between microcavities, noticeable  $Q$ -factor enhancement of one antibonding supermode is achieved without shifting the supermode wavelength (Fig. 5). Furthermore, such a PM demonstrates directional light emission, which cannot be achieved in isolated WG-mode microdisks (see inset to Fig. 5). Our studies also indicate that this directional emission pattern is preserved if the disk-to-disk distance is varied.

It should also be noted that other system parameters can be tuned to manipulate resonance wavelengths and  $Q$  factors of microcavities through mode

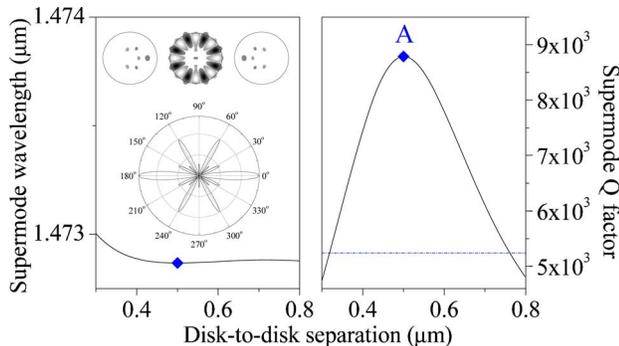


Fig. 5. (Color online) Resonance wavelength (left) and  $Q$ -factor (right) of an antibonding WG supermode in a three-disk PM. The central disk of radius  $1.065 \mu\text{m}$  is separated from the side disks of radii  $1.1 \mu\text{m}$  by  $400 \text{ nm}$  air gaps. Inset, supermode near-field portrait and directional far-field emission pattern at point A.

coupling at exceptional points. Among these are the refractive index of the cavity substrate and the size and/or position of a hole pierced in the cavity, which can be adjusted to enhance a WG-mode  $Q$  factor [14,15] or to achieve directional emission on a high- $Q$  WG mode [16].

In summary, a comprehensive numerical study was performed to elucidate the mechanisms of mode coupling in PMs with various degrees of cavity size mismatch. The study offers an alternative approach to designing novel PM-based components with improved functionalities. In contrast to PM structures composed of identical cavities that may require fabrication accuracy beyond the capabilities of modern technology, the proposed approach does not rely on precise cavity size-matching to achieve the desired device performance. This approach paves the way for new designs of more complex PM structures and arrays, which may eventually lead to new capabilities and applications in microphotonics and nanophotonics.

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